

Solving Polynomials

$$\textcircled{1} \quad x^3 + 3x^2 + 16x + 48 = 0$$

$$\textcircled{x^2} \textcircled{(x+3)} \textcircled{+16} \textcircled{(x+3)} = 0$$

grouping
b/c
4 terms

$$(x+3) \mid (x^2 + 16) = 0$$

$$x+3=0$$

$$x = -3$$

cut

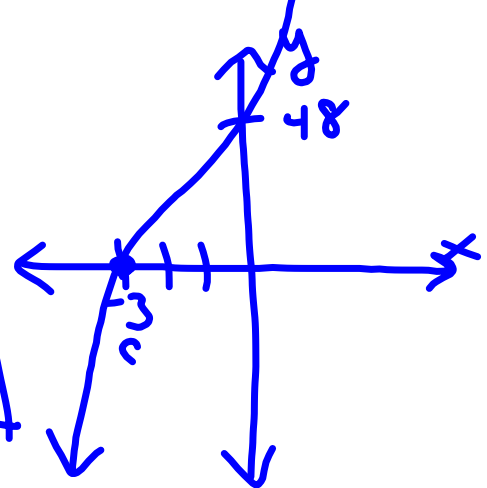
$$x^2 + 16 = 0$$

$$\quad \quad -16 \quad -16$$

$$\sqrt{x^2} = \sqrt{-16}$$

$$x = \pm 4i$$

EB: ↓ ↑



$$\textcircled{2} \quad x^3 + x^2 - 14x - 24 = 0$$

$$x^2(x+1) - 2(7x+12) = 0$$

can't factor!
 So, let's use calc. + syn. division
 "cheat"

$$x = -3$$

cut

$$\begin{array}{r|rrrr}
 -3 & 1 & 1 & -14 & -24 \\
 & \downarrow & -3 & 6 & 24 \\
 \hline
 & 1 & -2 & -8 & 0
 \end{array}$$

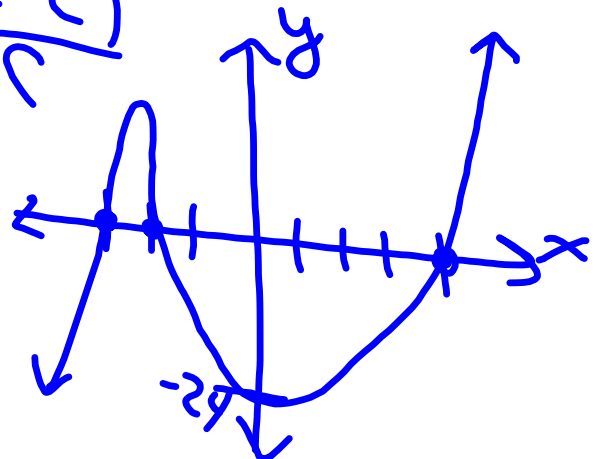
$$+x^3$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

EB: ↓↑

$$x = 4 \quad | \quad x = -2$$



$$\textcircled{3} \quad x^3 - 8x^2 + 11x + 20 = 0$$

$$x = -1$$

$$\begin{array}{r} -1 \overline{) 1 \ -8 \ 11 \ 20} \\ \underline{ 1 \ -9 \ 20} \\ 1 \ -9 \ 20 \ 0 \\ x^2 - 9x + 20 = 0 \\ (x-4)(x-5) = 0 \\ \hline 4 \ | \ 5 \end{array}$$

$$x = 4$$

$$\begin{array}{r} 4 \overline{) 1 \ -8 \ 11 \ 20} \\ \underline{ 4 \ -16 \ 20} \\ 1 \ -4 \ -5 \ 0 \\ x^2 - 4x - 5 = 0 \\ (x-5)(x+1) = 0 \\ \hline 5 \ | \ -1 \end{array}$$

$$x = 5$$

$$\begin{array}{r} 5 \overline{) 1 \ -8 \ 11 \ 20} \\ \underline{ 5 \ -15 \ 20} \\ 1 \ -3 \ -4 \ 0 \\ x^2 - 3x - 4 = 0 \\ (x-4)(x+1) = 0 \\ \hline 4 \ | \ -1 \end{array}$$

$$\textcircled{4} \quad 3x^4 - 2x^3 - 37x^2 + 24x + 12 = 0$$

$$\begin{array}{r|rrrrr} & 3 & -2 & -37 & 24 & 12 \\ (x-1) & \downarrow & 3 & 1 & -36 & -12 \\ \hline & 3 & 1 & -36 & -12 & 0 \end{array}$$

$$3x^3 + 1x^2 - 36x - 12 = 0$$

grouping

$$\textcircled{x^2} \textcircled{(3x+1)} \textcircled{-12} \textcircled{(3x+1)} = 0$$

$$(3x+1)(x^2-12) = 0$$

$$\begin{array}{l|l} 3x+1=0 & x^2-12=0 \\ \hline +1-1 & +12+12 \end{array}$$

$$\frac{3x}{3} = \frac{-1}{3}$$

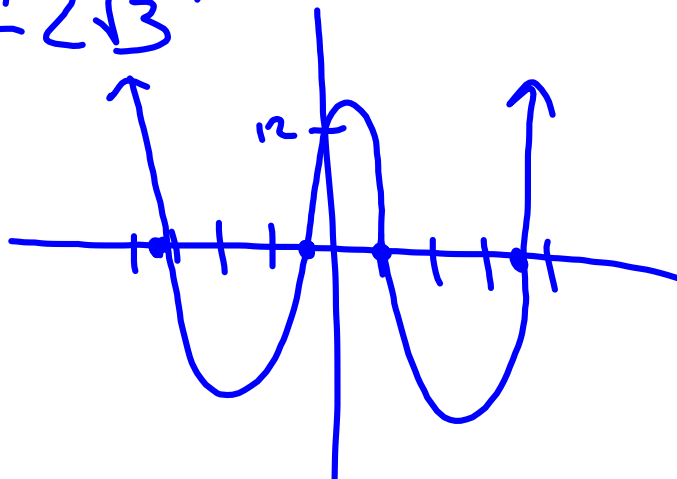
$$\sqrt{x^2} = \sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

roots:

$$x = 1, -\frac{1}{3}, \pm 2\sqrt{3} \approx \pm 3.5$$

EB: ↑↑



⑤ $4x^4 + 12x^3 + 9x^2 = 0$
 $x^2(4x^2 + 12x + 9) = 0$

Split the Middle
 $ac = 3b$

$b = 12$



$4x^2 + 6x + 6x + 9$

$(2x)(2x+3) + 3(2x+3)$

(x-0)²
 Bounce

$x(2x+3)(2x+3) = 0$

Bounce

$x^2 = 0$
 $x = 0$

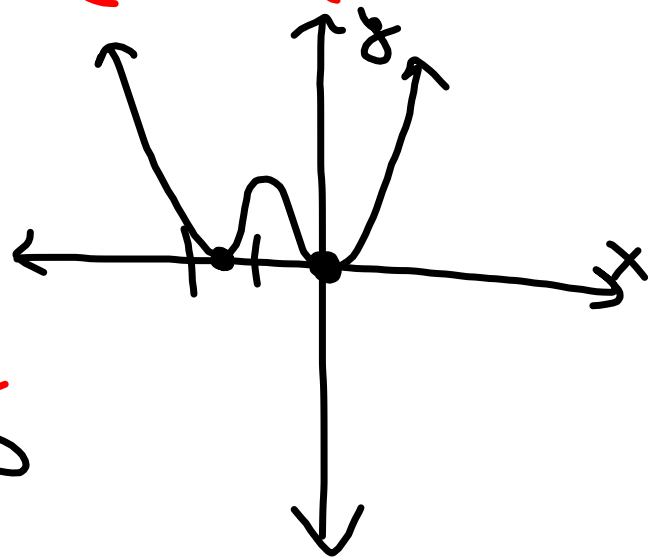
$2x + 3 = 0$
 $-3 - 3$

$x = -\frac{3}{2}$

$\frac{2x}{2} = \frac{-3}{2}$

$(2x+3)^2$

Roots: $x = 0, -\frac{3}{2}$
 B, B



$$\textcircled{6} \quad x^4 + 81 = 18x^2$$

$$x^4 - 18x^2 + 81 = 0$$

~~$$(x^2 - 9)(x^2 - 9) = 0$$~~

$$(x+3)(x-3)(x+3)(x-3) = 0$$

$$\begin{array}{c|c|c|c} -3 & 3 & -3 & 3 \end{array}$$

$$x = \pm 3$$

$$(x+3)^2(x-3)^2 = 0$$

